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Determination of Optimal Foreign Exchange Reserves in Nigeria

*Moses K. Tule, E.N. Egbuna, J.E.L. Sagbama, Abdusalam, S.A, Ogundele, O.S, A.O. Oduyemi
and S. Oladunni*

CENTRAL BANK OF NIGERIA

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Determination of Optimal Foreign Exchange Reserves in Nigeria

By

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Abstract

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This paper examines the optimal level of international reserves for Nigeria by minimizing the central bank's cost function. The study imposed the cumulative quarterly output loss associated with the global financial crisis and Nigerian Banking sector crisis between 2008Q1 and 2009Q4 (i.e. 52.80 per cent) and 2009Q1 – 2010Q4 (i.e. 32.22 per cent) as the maximum and minimum output losses for the entire study period. The study found that while actual reserves had been above the optimal reserves level between 2008Q1 – 2010Q4, the average core reserves available to the economy was however, insufficient to absorb the adverse economic impact of financial crises if they occur in future.

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Key words: International reserves, Sovereign risk, Optimization, GARCH, Cointegration

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1.0 Introduction

Economies maintain foreign reserves for different reasons which including amongst others to efficiently manage exchange rate volatility and adjustment costs associated with variations in international payments (Elhiraika and Ndikumana, 2007). Recently, there has been a growing trend in reserves accumulation amongst developing countries. The International Monetary Fund (IMF) estimates that the global external reserves holding increased from US\$1.57 trillion in 1996 to US\$11.69 trillion in 2013, with the share of developing and emerging economies increasing from US\$0.55 trillion to US\$7.87 trillion. The phenomenal rise in external reserves holding across many emerging markets and oil exporting countries in recent years have been motivated largely by the drive for self-insurance against adverse external shocks (Adam and Ndikumana, 2007).

Nigeria has witnessed significant rise in external reserves from US\$3.40 billion in 1996 to US\$28.28 billion in December 2005 peaking at an all-time high of US\$62.08 billion in September 2008 before declining to US\$ 39.07 billion as at July 2014 (Figure 1). The huge accretion to external reserves between 2000 and 2008, reflected favourable developments in the oil market; including high prices, strong demand and improved domestic production. However, the significant drop in reserves between 2008 and 2010 was attributed to the effects of the 2008/09 Global Financial Crisis (GFC), significant production declines due to insecurity in the oil producing region and high import bills. More recently, the effects of tapering in the US coupled with dwindling fiscal buffers, accentuated the threat of depletion of the country's external reserves.

Evidence suggests that the depletion in foreign reserves witnessed in Nigeria in recent times could elevate risk concerns among foreign investors. This could have serious implications for risk premium, portfolio flows, short-term external debt position, balance of payment position and economic growth. Also dwindling fiscal buffers tend to increase the country's reliance on foreign portfolio flows which are known to be volatile and characterized by sudden stop constitute a major risk to exchange rate stability, especially with uncertainties around capital flows and oil price. This suggests that a country's ability to manage its short-term obligations to the outside world, maintain a disciplined fiscal regime and attract long-term capital is crucial in the determination of its risk premium (Ozyildirim and Yaman, 2005).

The debate on what constitutes an optimum reserve holding remains unsettled in the literature. While some countries have remained aggressive in the accumulation of external reserves, others strive to maintain adequate reserves based on certain international standards. Practical experience suggests at least three import cover² "rule of thumb" in determining the optimal level of reserves (Mendoza 2004). Import based reserve adequacy criteria suggests that 30 per cent of broad money or 4 months of import covering reserves can be considered as a minimum benchmark for reserve

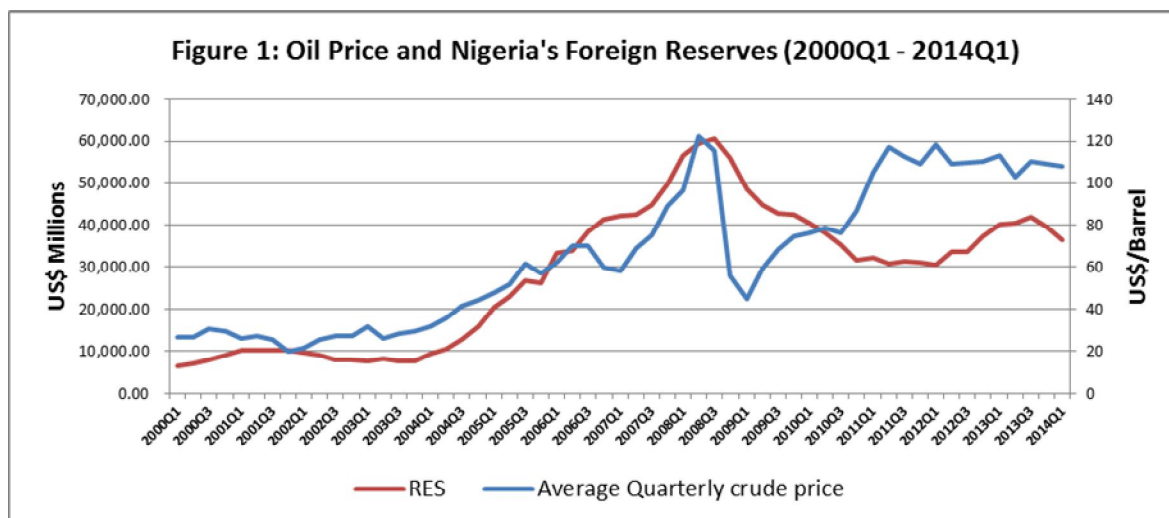
² Import cover in the literature is defined as the ratio of average monthly import to the average stock of foreign reserves. The inverse of which is the reserves to imports ratio.

adequacy. Similarly, Wijnholds and Kapteyn (2001) proposed that countries on managed float or on fixed exchange rate regime could maintain reserves to cover around 10 and 20 per cent of broad money, while the IMF posits 3 months of import cover. The role of reserves in macroeconomic management remains debatable, as both low and high reserves portfolios have their characteristic cost implications. The conventional external reserves adequacy ratios may not represent optimality in external reserves holdings. Therefore, it is important to estimate the optimal external reserve holding, while taking cognizance of adverse external shocks, cost profile of reserve maintenance and sensitivity of international capital to macroeconomic fundamentals. This would facilitate the comparison of the optimal trend with the conventional benchmarks, and help determine if actual reserves are beyond or below the optimal levels in which case, the country could be incurring some costs or benefits.

The knowledge of how a country's sovereign risk may be impacted by key external and fiscal variables such as portfolio flows, fiscal deficit and short-term external debt in relation to the foreign reserve levels and output is critical for the attainment and sustenance of macroeconomic stability. More importantly, identifying the external reserves level which is deemed optimal to enable the country adequately absorb the effect of a severe adverse shock is key to effective macroeconomic management. The objective of this paper is to estimate the optimal external reserve level for Nigeria. Following this introduction, Section 2 presents some stylized facts while Section 3 examines related literature including theoretical framework. Section 4 discusses data and methodology. Section 5 interprets the estimation result for the empirical analysis, while section 6 concludes with policy recommendations.

2.0 Stylized Facts

Nigeria's external reserves derive mainly from the proceeds of crude oil production and sales. The main sources of rising external reserves in Nigeria include inflows of oil revenues complemented by diaspora remittances, growing foreign direct investment (FDI) and portfolio investments, capital inflows, banks' on-lending activities to foreign financial institutions, growing guarantees and grants, etc. Nigeria's external reserves rose phenomenally from 2005 and maintained the upward trend until the wake of the global financial crisis when it nose-dived from its peak in 2008. From an average position of \$6.32 billion between 1990Q1 and 2004Q4, the external reserves peaked at \$62.08 billion in 2008Q3. It, however, declined to its 2014Q1 position of \$38.33 billion (Chart 1).



Source: Statistics Database, CBN (2014)

Table 1: Average periodic trend in Interest Rate Spread, FPI, and Foreign Reserves (2000Q1 – 2013Q3)

	2000q1 - 2006q1	2006q2 - 2007q4	2008q1 - 2010q4	2011q1 - 2014q1
Average Spread	3.05%	1.54%	3.96%	8.96%
Average FPI	135.03	694.69	457.93	3,098.37
Average reserves	12,635.54	42,493.63	46,843.65	38,176.10

Source: SD, CBN & authors' calculations

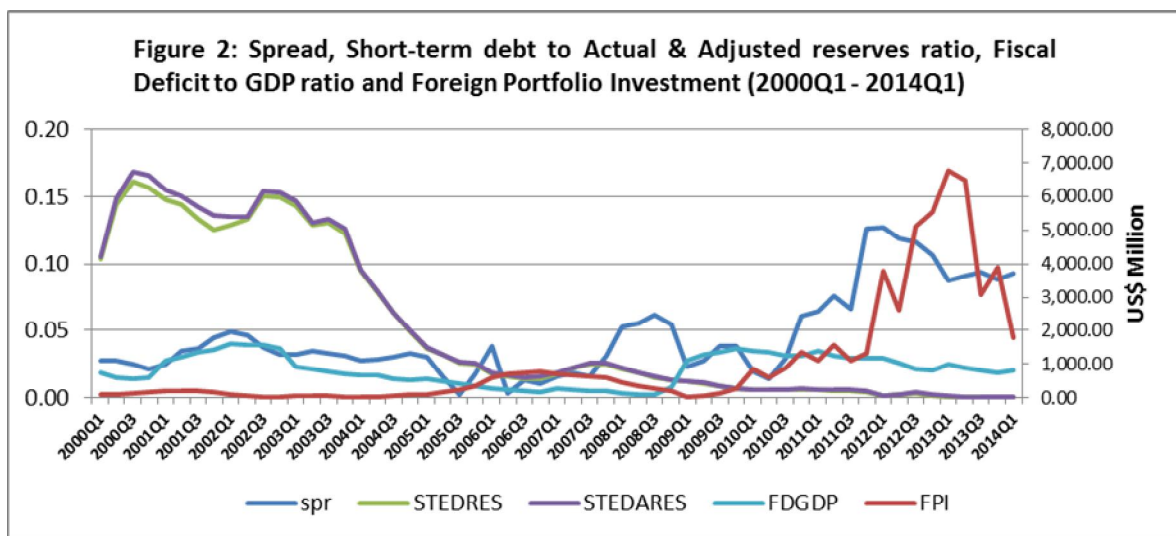
Table 1 indicates that between 2000Q1 and 2006Q1, reserves, FPI and interest rate spread averaged US\$12.64 billion, US\$135.03 million and 3.05 per cent, per quarter. This period was characterized by high levels of short-term debt to reserves ratio, which adversely impacted the inflow of FPI. Between 2006Q2 and 2007Q4, reserves, FPI and interest rate spread averaged US\$42.93 billion, US\$694.69 million and 1.54 per cent, per quarter. The increase in FPI despite the lower spread could be explained by the significant decline in the ratio of short-term debt to reserves during the period. Furthermore, it coincided with a period when the economy exited the Paris and London club debt obligations. The significant increase in the stock of reserves was primarily as a result of the steady increase in crude oil prices, during the period.

The period 2008Q1 and 2010Q4 saw a decline in the average FPI to US\$457.93 million, despite having an average interest rate spread of 3.96 per cent. This was primarily due to the onset of the global economic crisis, which also impacted the Nigerian economy, triggering the Nigerian Banking crisis. Furthermore, the crises triggered the withdrawal of credit lines and capital flow reductions as foreign investors repatriated funds back to their home countries to shore up their balance sheets. Significantly, despite the reduction in FPI, reserves averaged US\$49.30 billion during the period. The period also witnessed a significant drop in reserves from its peak of US\$62.08 billion in 2008Q3 to US\$33.00 billion in 2010Q4.

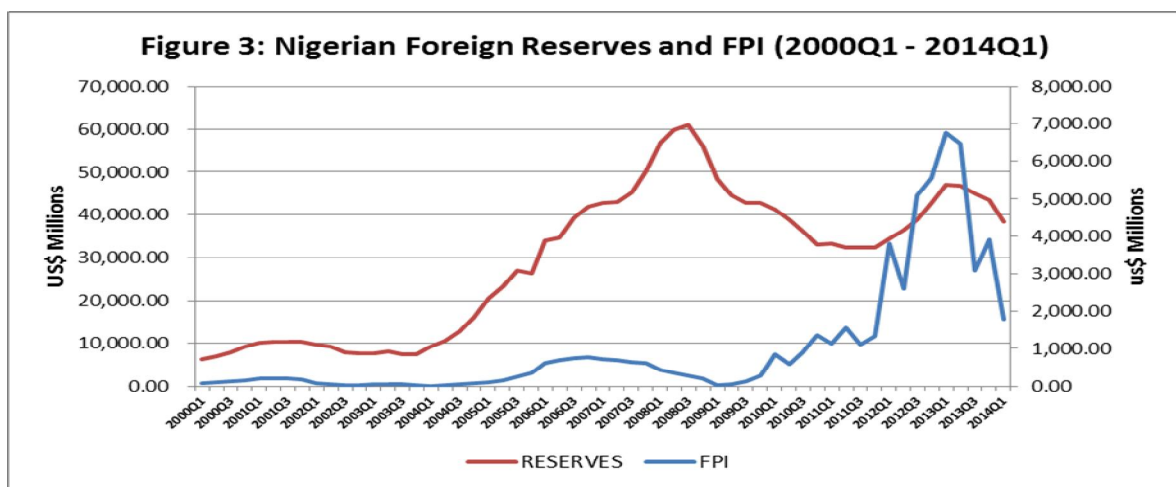
Between 2011Q1 and 2014Q1, average FPI and interest rate spread increased to US\$3.10 billion and 8.96 per cent, respectively, while average foreign reserves declined to US\$38.18 billion. The increase

in FPI was triggered by the massive increase in average interest rate spread³ and improved fundamentals of the Nigerian economy, occasioned by steady GDP growth and stable prices. Furthermore, though average reserves were lower than that of the period 2008Q1 to 2010Q4, the average for the period was higher than the low point of US\$33 billion in 2010Q4.

Figures 2, 3 and 4 suggest that between 2000Q1 and 2005Q3, FPI was primarily influenced by the uncertainty about capacity to service the short-term sovereign debt which was signaled by the high short-term external debt to foreign reserves ratio. Furthermore, between 2010Q3 and 2014Q1, FPI was strongly influenced by the interest rate spread and good macroeconomic fundamentals, which was manifested by strong GDP growth and stable prices. Significantly though, the major driver of foreign reserves remained the international price of crude oil.

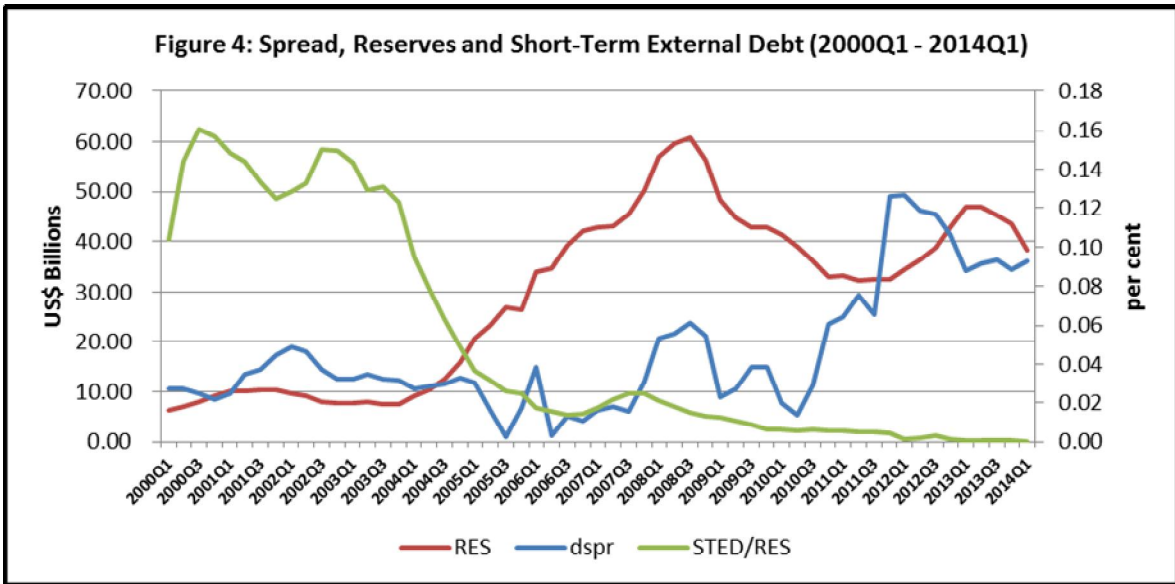


Source: SD, CBN & authors' calculations



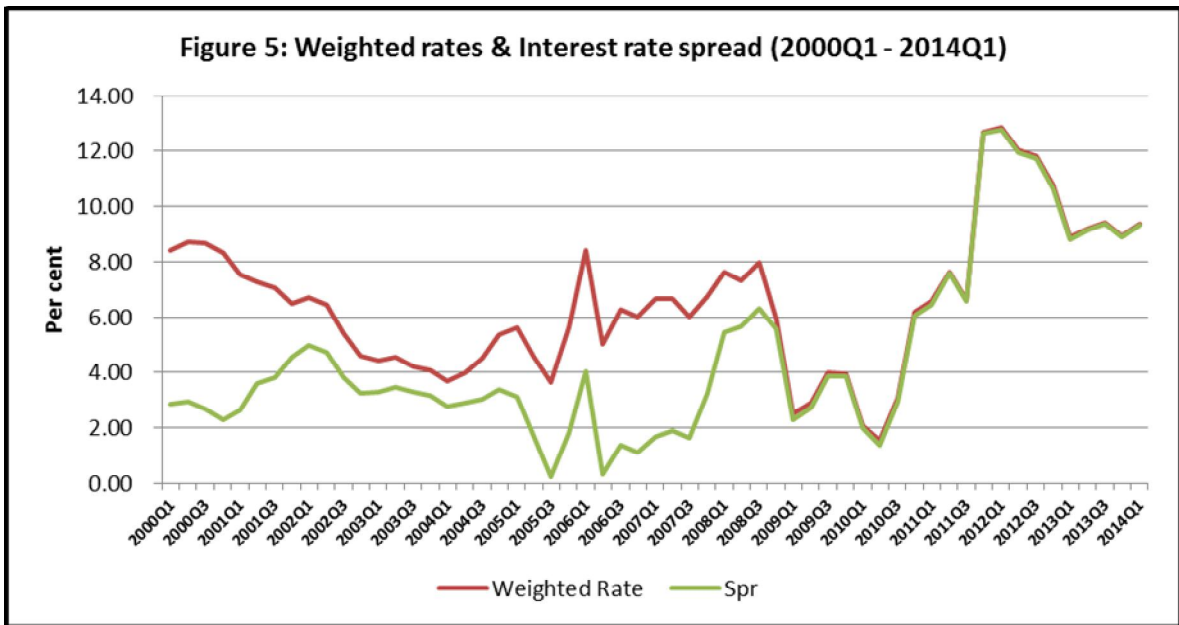
Source: SD, CBN

³ The spread is the difference between weighted return on Nigerian sovereign debt instruments and 90-day FED NTB rate



Source: SD, CBN & authors' calculations

Figure 5 shows that prior to the onset of the global economic crisis, the interest rate spread was below the weighted average rate, but it has subsequently mirrored the weighted interest rate on Nigerian sovereign debt instruments, with the collapse of the external cost of funds to near zero per cent.



Source: SD, CBN & authors' computations

3.0 Literature Review

The literature on the determinants of optimal level of reserves revolves around three central areas of consensus. In the first cluster of literature, Heller (1966), Frenkel and Jovanovic (1981)⁴ viewed foreign reserves accumulation as a process of satisfying the obligation of external payments and suggest a framework of ratio of reserves to imports. This establishes whether a country has the minimum capability to support its external obligations. Triffin (1960) suggests import based reserve adequacy of 30.0 per cent of Broad Money (M2) or 4 months of import covering reserves. Similarly, Wijnholds and Kapteyn (2001) proposed that countries can maintain reserves to cover around 10.0 and 20.0 per cent of broad money if operating a managed float or fixed exchange rate regime.

The second consensus consist of contributions from authors like, Calvo (2002), Rodrik and Velasco (1999), Bird and Rajan (2003), Garcia and Soto (2004), Jeanne and Ranciere (2011), ECB (2006) etc. They argue that the maintenance of reserves at levels other than at the optimal level could trigger a macroeconomic disequilibrium. Calvo (2002) noted that, the accumulation of foreign exchange reserves leads to monetary expansion and hence inflation. This is, however, in contrast to the submission of Victor and Vladimir (2006), who argued that reserves are accumulated through maintenance of government budget surplus, which averts inflationary pressure. Their study discovered that there was no link between the accumulation of foreign reserves and inflation. Others, like Jeanne and Ranciere (2011), suggest that reserves are deployed to fill balance of payments gap associated with GDP losses arising from external shocks and sudden restrictions⁵ in accessing international capital. Bird and Rajan (2003) acknowledge that the desire to maintain reserves at adequate levels help ensure that interest rates are kept at competitive levels to discourage capital outflows. Dooley et al. (2003) posit that reserves are accumulated to facilitate the actualization of the macroeconomic agenda of government such as export oriented growth and job creation.

The third is the optimizing reserves approach argument. Ben-Bassat and Gottlieb (1992) posits that reserves are held at levels in which the added benefits and costs of keeping reserves are equated. It posits that reserves accumulation has the benefit of signaling a low default risk on sovereign debt, which translates into improved sovereign risk rating and lower compensatory interest premium to compensate international investors for absorbing the risk from investing in the country's sovereign debt instruments. Furthermore, there is an associated opportunity cost of keeping reserves, which includes the income forgone for not investing the reserves in higher interest income earning instruments, output losses and interest expense from the finance of expenditure through taxes and

⁴ Recent examples in this literature include Wijnholds and Kapteyn (2001) and Barllan et al. (2007).

⁵ Similarly Ben-Bassat and Gottlieb, 1992, Aizenman and Lee, 2007, Raphael Espinoza (2014), etc., argument on optimal reserves suggest that reserve accumulation can be viewed as self-insurance to mitigate and prevent an undesired output drop or the crisis caused by sudden stops.

debt rather than depletion of the reserves. Intrinsicly, the approach ascertains that level of optimal reserves that either minimizes the cost or maximizes the benefit of keeping reserves, by solving the Central Bank's optimization problem. Heller (1966) posits that optimal reserves holding is achieved when the marginal cost and benefits of holding reserves are equated. The approach derives an expression for optimal reserves as a function of observables (such as the level of imports and external debt) and a few unknown quantities, namely: the opportunity cost of holding reserves, the output cost of default, the probability of default and the effect of higher reserves on this probability. The outcomes of the literature differ across countries.

Prabheesh (2013) empirically determined the optimal level of international reserves for India by explicitly incorporating the country's sovereign risk associated with default of external debt due to financial crisis. The empirical result shows that the volatility of foreign institutional investment, short-term debt to reserves and the fiscal deficit to GDP significantly explains the variations in risk premium. The study suggests that international reserves in India are higher than the estimated optimum level of reserves. Suheyla and Yaman (2005) study on optimal reserves and its adequacy in Turkey between 1998 and 2002 indicated that actual reserves were below the optimal and adequate levels, when a cumulative GDP loss in excess of 5.0 per cent during a financial crisis is imposed on the entire sample period. Jeanne and Rancière (2006) argue that reserves allow the country to smooth domestic absorption in response to sudden stops, but yield a lower return than the interest rate on the country's long-term debt..

The literature on developing and low income countries has been less extensive. Abiola et al (2013) study of the demand for reserves in Nigeria concluded that Nigeria's foreign reserve were adequate, based on international benchmarks. David and Yaaba (2012) used an Autoregressive Distributed Lag (ARDL) to estimate the determinants of foreign reserves in Nigeria and found strong evidence in support of income as a major determinant of reserves management in Nigeria. Oputa and Ogunleye (2010) indicated that while Nigeria's reserves were adequate based on international standards, actual reserves were on average below their estimated adequate reserves. They concluded that the economy needed to sustain reserves accumulation to enable it adequately absorb the adverse impact of external shocks (e.g. financial crises and slump in oil prices). Udo and Antai (2014) suggest that reserves accumulation in Nigeria had an adverse impact on investment and economic productivity, and recommended a cut back in reserve accumulation to finance domestic investment.

The central message of the different approaches and studies is that the motives for keeping reserves determine the key variables which influence actual reserves levels; however, there is an optimal level of reserves that engenders macroeconomic stability even in the presence of a financial crisis. The study is built on Ben-Bassat and Gottlieb (B-G, 1992) framework and Prabheesh (2013), given the simplicity in estimating optimal reserves and the macroeconomic peculiarities of the Nigerian

economy (i.e. monoculture – dependence on the oil sector for foreign revenue receipts and high import dependence).

4.0 Theoretical Framework, Data and Methodology

The study adopts the framework developed by Ben-Bassat and Gottlieb (B-G, 1992), which was employed by Prabheesh (2013) to determine optimal international reserves in India between 1994 and 2008. The B-G model is based on the cost-benefit approach in determining optimal reserves. Theoretically, optimality is achieved by equating the marginal cost of holding reserves to the marginal benefits of doing so. The cost of holding reserves is identified as the potential income forgone for holding foreign exchange in reserves rather than utilizing it in other purposes, with economic benefits, while the benefit is the avoidance of output losses associated with Balance of Payment (BOP) and exchange rate fluctuations. Economies by virtue of international trade and finance are susceptible to global economy shocks, which impact their stock of foreign reserves and international value of the local currency. Thus, every economy has a probability (π) that such shocks will result in reserves depletion. Alternatively, (π) is the probability that the economy may default of its debt obligations faced with adverse financial and economic shocks.

$$\pi = f(R, Z) \tag{1}$$

$$\frac{\partial \pi}{\partial R} = \pi < 0^6 \text{ (i.e. convex)} \tag{1.1}$$

Where:

R is stock of reserves holdings and Z is a collection of economic variables, which influence reserves depletion and the ability of the economy to meet her debt obligations as and when due (i.e. default risk).

Based on the above discussions, the expected total cost to the economy for holding reserves is:

$$E(TC) = \pi C_0 + (1 - \pi) C_1 \tag{2}$$

Where:

E is the expectation operator,

TC is the total cost of reserves holding to the economy

C_0 is the cost of holding low reserves (i.e. potential output loss)⁷

C_1 is the total opportunity cost for holding reserves

$$C_1 = rR \tag{2.1}$$

⁶ $\partial(\pi)/\partial R = \pi_R < 0$, because an increase in reserves improves the ability of the economy to repay its debt obligations, absorb adverse economic shocks, and moderate volatility in BOP and foreign exchange rate.

⁷ This is the difference between potential GDP and actual GDP

r is the interest forgone for holding reserves (i.e. interest rate that would have been earned, if the reserves were converted into domestic currency and invested in Treasury Bills) and R is stock of reserve holdings.

Substituting (2.1) into (2) we get equation (2.2)

$$E(TC) = \pi C_o + (1 - \pi)rR \quad (2.2)$$

Optimality requires the minimization of the expected total cost of reserves to the economy (i.e. optimality in reserves accumulation is obtained when the cost of reserves accumulation is at its minimum level).

Taking 1st order derivative of equation 2.1 with respect to R and equating it to zero we have:

$$\frac{\partial E(TC)}{\partial R} = (C_o \frac{\partial \pi}{\partial R} - \frac{rR \partial \pi}{\partial R} + (1 - \pi_{rz})r = 0 \quad (3)$$

Substituting (1.1) into (3) we get equation (3.1)

$$\frac{\partial E(TC)}{\partial R} = \pi_R(C_o - rR) + (1 - \pi_{rz})r = 0 \quad (3.1)$$

Solving for optimal reserves holding R^* , we get equation (4)

$$R^* = \frac{(1 - \pi)}{\pi_R} + \frac{C_o}{r} \quad (4)$$

Once the first order condition is established, the individual parameters are estimated (i.e. C_o [the output gap using Hodrick-Prescott (1981) filter method], π_{LZ} [probability of default] and $C = rR$ [i.e. opportunity cost of holding reserves]).

4.1 Key Concepts

4.1.1 Cost of Default

In line with the methodology adopted by Ozyildirim and Yaman (2005), Prabheesh (2013), estimated the adjustment cost as the potential output loss due to the prospect of insolvency and financial crisis. It takes into cognizance the fact that low reserves may threaten the ability of an economy to meet its debt obligations in a period of crisis. The country's credit rating is likely to drop and may be unable to secure credit and credit lines to meet up its commercial and financial obligations. Consequently, the economy would be operating below its pre-crisis capacity and thus be on a lower growth trajectory during the crisis and immediate post-crisis periods. In view of this, the cost of default on external debt or cost of insolvency is good proxy for the cost of reserves depletion in

developing economies which are characteristically borrowing economy with sustained BOP disequilibrium⁸.

4.1.2 Opportunity cost (C₁)

This refers to the income forgone for holding reserves and is computed as the product of domestic returns on 91-day Treasury bill and total reserves.

4.1.3 Probability of Default (Π_{r,z})

The Prabheesh (2013) study estimated the probability of default function as:

$$\Pi_{r,z} = \frac{e^f}{1+e^f} \quad (5)$$

Where:

f is a function of economic variables that determines the likelihood that an economy will default on its sovereign debt obligations. Hence f is defined as the odds of default $\left(\frac{\Pi_{r,z}}{1-\Pi_{r,z}}\right)$ which under a perfect capital market is equivalent to the discounted risk premium (i.e. $\left(\frac{i-i^*}{1+i}\right)$),

Where:

i = rate of return on a risky financial assets (e.g. borrowing rate)

i^* = rate of return on risk free assets (e.g. sovereign bonds)

Equation (5) is derived based on the preceding discussions

$$\left(\frac{\Pi_{r,z}}{1-\Pi_{r,z}}\right) = \left(\frac{i-i^*}{1+i}\right) \quad (6)$$

The discounted risk premium $\left(\frac{i-i^*}{1+i}\right)$ in a perfect international capital market can be interpreted as the spread between returns on investment in domestic financial instruments and returns on safe (risk free) international financial instruments (e.g. LIBOR, T-Bills etc.). This in effect measures/proxies the sovereign risk of a nation. A high positive spread is indicative of high risk premium attributable to the poor sovereign rating of the economy. International investors are thus likely to demand a higher spread to encourage them to invest in domestic financial instruments.

Substituting (5) into (6) and taking logs of the left and right hand sides we have:

⁸ Prabheesh (2013) imposed an output loss of 4.8 and 7.5 per cent of GDP on the model for the sample period (1995 – 2009), based on the potential output loss to the Indian economy between 1991 and 1994, resulting from the economic crisis of 1991 – 1994.

$$f = \log\left(\frac{\Pi_{rz}}{1 - \Pi_{rz}}\right) = \log\left(\frac{(i - i^*)}{(1 + i)}\right) = \log(e^f) \quad (7)$$

Based on previous discussions, f is equal to the log of discounted risk premium or spread and is function of the economic variables collectively captured by Z .

$$f = f(Z) \quad (8)$$

Thus, f can be estimated by regressing it with macroeconomic fundamentals. Following Edwards (1986), Nogue's and Grandes (2001), Ferrucci (2003), Ozyildirim and Yaman (2005) and Prabheesh (2013), the risk premium equation can be specified as follows:

$$\log\left(\frac{i - i^*}{1 + i}\right) = a_0 + a_1 \log(sii) + a_2 \log\left(\frac{sted}{rr}\right) + a_3 \log\left(\frac{fd}{gdp}\right) + \varepsilon_t \quad (8.1)$$

The economic variables contained in the set Z , which explained the variations in the spread are as follows:

1. **Volatility of foreign institutional investment** – This captures the impact of the volatility of foreign international investment on risk premium. It takes cognizance of the characteristically mobile nature of international capital and the relationship between macroeconomic risks and the interest premium on international borrowings. Consequently, it is expected that a positive relationship exists between foreign institutional investment and the spread, because investors must be paid a high premium to compensate them for investing in debt instruments in a risky macroeconomic environment.
2. **Short-term external debt to reserves** – This helps explain how the ability of the government to service short-term debt obligations through reserves depletion impacts on risk premium. The capacity to meet immediate debt obligations is influenced by existing stock of actual and core⁹ foreign reserves at the disposal of the government. The larger the stock of reserves, the greater the ability to meet short-term obligations and thus, the lower the risk of default. This translates into low default risk and low compensatory risk premium to international investors for absorbing the risks associated with investing in short-term debt instruments issued by the government. This portends an expected positive relationship between the ratio and the spread.
3. **Fiscal Deficit to GDP** – This essentially captures the inefficiency of government in managing its operations economically and efficiently. Persistently, high fiscal deficit/GDP ratio, puts into doubt the ability of the government to raise sufficient funds to meet its debt obligations as and when due. Consequently, investors in sovereign debt instruments issued by the government would require a high premium to compensate them for absorbing a high risk of possible default on debt obligations by the issuing government. A high ratio signals national

⁹ The core reserve can be defined as total reserves less the quarterly adjusted cumulative foreign portfolio investment. This gives an indication of the unencumbered reserves available to the authority to defend the currency in the event of capital flight.

incapacity to service debt obligations as they mature and translates to lower credit worthiness thus, requiring high compensatory premiums for investors willing to absorb such risks. Thus, it is expected that a positive relationship exists between the interest rate premium and the ratio of short-term debt to external reserves.

4.1.4 Volatility of Portfolio Investment

The Study established the volatility of portfolio investment with an Autoregressive Conditional Heteroscedasticity (ARCH)/ Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, to justify its inclusion as a variable in the determinant of the discounted risk premium¹⁰.

The Arch (p) model utilized is given below:

$$S_{it} = \mu + \varepsilon_t \quad (9)$$

Where S_{it} is sentiment of international investors

$$\frac{\varepsilon_t}{\Omega_{t-1}} \sim N(0, h_t) \quad (10)$$

Where μ is the mean of s_{it} , the error term is conditional upon the set of information Ω , that existed in the immediate past period $t-1$. The error term ε_t is normally distributed and has a mean and variance of zero and h_t , respectively. Equation (10) is the conditional mean equation for s_{it} .

$$h_t = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 \quad (11)$$

$\omega > 0; \alpha_1, \dots, \alpha_p \geq 0$

The Variance equation (11) indicates that the variance h_t is conditional on the mean ω and available set of information on the variance of past periods ε_{t-i}^2 .

4.2 Data

The study utilizes quarterly data covering the period 2000Q1 – 2014Q1. Data for key economic variables of interest were sourced from the following:

- CBN's statistical database, Statistical Bulletin and internally generated data from relevant departments of the CBN. The variables were 90-day T-Bill rates, stock of foreign exchange reserves, foreign portfolio investment, nominal import, Naira-US\$ exchange rate, nominal GDP.
- The World Bank database – the data obtained from the source was the US 90-day Treasury bill rates.
- Bloomberg terminal – 90-day LIBOR rate was obtained from this source.

¹⁰ See Prabheesh (2013)

The variables GDP, FPI and Fiscal Deficit were converted to United States of American Dollars (US\$) using the average prevailing official exchange rates for the quarter. In so doing, it internalized the fluctuations in foreign exchange rate into the variables and by reasoning, internalized the impact of foreign exchange movements and its impact on reserves within the model.

The variables of interests, rationale for their inclusion for the Nigerian environment and their computations are described below:

- i. **Spread** – The spread which denotes risk premium is derived by subtracting interest rates on US short-term instrument from the weighted interest rates. The computed weighted interest rate is derived from both the London Interbank Bid Offer Rate (LIBOR) and the Nigerian Treasury Bill (NTB) rates. The rationale for this is that the national debt comprises the domestic and external portion. The NTB rate serves as a good proxy for the cost of the domestic portion of the national debt, while LIBOR serves as a good proxy for the cost of external borrowing as the bulk of commercial lines and Eurobonds issued by the FGN and private corporations have their prices linked to the LIBOR. The rates are weighted by the ratio of domestic and external debts to total national debt. Thus, the weighted rate is derived as:

$$I_w = \alpha_{DD}NTB + \beta_{ED}LIBOR \quad (12)$$

Where

- I_w = Weighted Interest rate,
 - α_{DD} = ratio of domestic national debt to total debt (domestic debt/total debt)
 - β_{ED} = ratio of external national debt to total debt (external debt/total debt)
- ii. **Sentiment of International Investors (Sii) (i.e. Foreign Portfolio Investment)** – International capital flows are characteristically mobile and can abruptly change direction of flow in response to sentiments about developments in an economy. This very peculiar nature of capital flows makes it a germane variable of interest in the model. The variable is of particular interest because Nigeria is a mono-product economy, which generates about 90 per cent of foreign exchange export revenue from the oil and gas sector. This is the major avenue through which reserves are built. External shocks, which trigger capital flow reversal invariably, would exert pressure on the reserves. This is due to supply side constraints in ramping up oil production to generate sufficient foreign exchange to moderate the impact of capital reversal on the country's foreign reserves. It is therefore important to understand how foreign portfolio investors react to the perceived state of the economy. As previously discussed, the state of the economy can be measured as the sovereign risk of the country, which is approximated by the discounted risk premium (i.e. spread). It is expected that a high spread, indicative of high sovereign risk, would elicit volatile movement in capital as investors would want to make quick gains and exit the economy before the situation changes. Consequently, the expectation is that a positive relationship exists between the volatility of portfolio investment and the spread (i.e. discounted risk premium).

- iii. **Short-term Debt to External Reserves** – The ability of a nation to service its immediate debt obligations can be ascertained from the quantum of reserves at its disposal. Invariably, a high level of foreign exchange reserves is indicative of ability to meet short-term debt obligations even in the face of severe adverse external shocks. This would translate to a low short-term debt to reserves ratio indicating a minimal encumbrance of reserves towards immediate obligations. Likewise, the low ratio would provide positive signals to potential investors and this is expected to improve the sovereign credit rating of the country and thus command a lower spread.

The inclusion of this variable stems from the peculiar import dependent nature of the economy. A significant part of trade transactions is carried out through trade credit lines between domestic and international financial institutions. In the event of a crisis these credit lines are typically first hit as international financial institutions call back their facilities and in some instances close those lines, to improve their balance sheet positions. Importantly though, the Nigerian debt crisis of the 1980's, that left the economy with a debt overhang of about US\$38 billion in 2004¹¹, was due to the crystallization of short-term debts and the inability of Nigerian merchants to meet up their debt obligations. Consequently, these were consolidated and taken over by the Nigerian government. Furthermore, domestic Deposit Money Banks (DMBs) had a torrid experience keeping trade lines open during the 2008 global economic crisis, as corresponding banks withdrew some of their credit lines. Consequently, the ratio provides an early warning signal on the state of the economy, which would invariably impact on the country's international credit rating, which in our study is proxied by the discounted risk premium (spread).

Apart from using the actual reserves in computing the short-term debt to reserves ratio, the paper opted to deduct the quantum of portfolio inflows from the stock of reserves. This was done because the discounted ratio gives a better indication of the ability of the economy to meet short-term obligations, in view of the mobility of international capital. This is particularly important because unlike short-term debt obligations which have contractual terms that determine their due dates, portfolio investments do not face similar constraints and can be called upon immediately, particularly in times of crisis. Thus reducing the actual reserves by the adjusted stock of quarterly portfolio inflow provides a clearer estimate of the available "core" reserves to defend the Naira in the event of a capital flow reversal.

Consequently, it is expected that a positive relationship exist between the discounted risk premium and the ratio of short-term external debt obligations to residual reserves. The short-term debt to external reserves is thus computed as:

¹¹ Source: <http://www.indexmundi.com/facts/nigeria/external-debt-stocks>

$$\text{Stedres} = \left(\frac{pn_t}{res_t} \right) \quad (13)$$

Where:

- Stedres = short-term debt to reserves ratio,
- Pn_t = promissory notes at time t, and
- Res_t = foreign reserves at time t.

The logarithm of the computed short-term debt to external reserves ratio was used in the analysis.

- iv. **Short-term Debt to Adjusted External Reserves** –The short-term debt to adjusted external reserves is computed as:

$$\text{Stedares} = \left(\frac{pn_t}{(res_t - afpi_t)} \right) \quad (14)$$

Where:

- Stedares = short-term debt to adjusted reserves ratio,
- Pn_t = promissory notes at time t,
- $aFPI_t$ = adjusted foreign portfolio investment at time t and
- Res_t = foreign reserves at time t.
- Adjusted or core reserves = $(res_t - afpi_t)$ (15)

The logarithm of the computed short-term debt to adjusted external reserves ratio was used in the analysis.

- v. **Fiscal Deficit to Nominal GDP ratio** – Fiscal deficit is an indicator of the Government's ability to manage its finances to deliver on its key mandate to the people. A high deficit to GDP ratio indicates that the government is unable to fund its current expenditure from its operations and thus may require debt to close the funding gap. If this is sustained it signifies heavy reliance on debt for the sustenance of governance; and signal a high probability of the future occurrence of debt overhang problems in the economy. A sustained high ratio will invariably reduce the credit worthiness of the economy as it signals a high risk of default on debt. Significantly, the risk of default becomes even more heightened during a crisis, and a high ratio would signal the inability of the government to meet its debt obligations in the event of external shocks.

This ratio like the previous two discussed are indicative of the health of the economy in the estimation of potential international investors; a high ratio indicates inability of government to meet debt obligations, investing in debt instruments in the economy would involve the absorption of significant risk, and would require a high compensatory premium. Consequently, it is expected that there exist a positive relationship between the discounted

risk premium and the ratio of fiscal deficit to GDP. Baldacci et al 2008 suggest that the sovereign risk rating and thus the risk premium of emerging economies is a function of their fiscal balance. The Fiscal deficit to nominal GDP ratio was computed as:

$$FDGDP_t = \left(\frac{fd_t}{ngdp_t} \right) \quad (16)$$

Where:

- $Fdgdpt$ = fiscal deficit to nominal GDP ratio,
- fd_t = fiscal deficit at time t, and
- $ngdp_t$ = nominal GDP at time t.

In applying the methodology in the estimation processes, the paper was able to obtain the probability of default and compute the optimal reserves during the study period. Furthermore, the paper is expected to ascertain the optimal core reserves level which the economy must maintain in order to adequately absorb the output loss associated with a severe external shock.

4.3 Methodology

In order to obtain the optimal reserves (R^*) the cost of default (C0), i.e., output contraction due to financial crisis, was calculated by employing the H-P filter method, while the ARCH model was used to derive the volatility series of foreign institutional investment (fii). The Multivariate Cointegration procedure was used for the estimation of the spread.

4.3.1 HP Filter Method for the estimation of the Cost of Default

This estimates a smoothed GDP series (i.e. GDP^T) that minimizes its variance around GDP^T , while subjecting the second derivative of GDP^T to a penalty constraint. The optimal GDP^T is that which minimises the following equation:

$$\sum_{i=1}^n (gdp_t - gdp_t^T) + \lambda \sum_{t=2}^{n-1} (\Delta gdp_{t+1} - \Delta gdp_t^T)^2 \quad (17)$$

Where n is the sample size and λ is the smoothing parameter. The cost of default or output gap is the difference between the computed smoothed gdp_t^T series and actual gdp_t series.

Following other studies (Prabheesh, 2013) the paper adopted the H-P filter method to estimate potential output loss associated with the 2008 global economic crisis. The cost of default used in the paper is the estimated output gap (i.e. percentage of output forgone) associated with the BOP crisis of 2008 – 2010, which was imposed throughout the sample period as the cost of reserve. The opportunity cost of holding reserves was estimated as the product of domestic returns on Nigerian 91 days T-Bill and total reserves (Prabheesh, 2013).

4.3.2 ARCH/GARCH Model for the Estimation of Volatility of Portfolio Investment

A generalized ARCH (GARCH) (P, Q) model was used to estimate the volatility of investor sentiments in Nigeria using weekly data on portfolio investment in bonds and equities, obtained from weekly reports on foreign capital inflows by investment type. The GARCH model had the same mean equation and distribution as (10) [i.e. $S_{it} = \mu + \varepsilon_t$], (11) [i.e. $\frac{\varepsilon_t}{\Omega_{t-1}} \sim N(0, h_t)$] while, the conditional

variance is of the GARCH type and is presented in equation (18) below

$$h_t = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i h_{t-i} \quad (18)$$

The Variance equation (13) indicates that the variance h_t is conditional on the mean ω , available set of information on the variance of past periods ε_{t-i}^2 , and the values of the past variance.

The condition that $\omega > 0$; $\alpha_1, \dots, \alpha_p \geq 0$ is maintained. The confirmation of the volatility of S_{it} (FPI), informed its inclusion in multivariate cointegration equation in the estimation of the discounted spread equation.

4.3.3 Multivariate Cointegration Estimation of the Discounted Risk Premium

Using the Johansen cointegration approach, the paper estimated the risk premium equation. The matrix form of the equation estimated is specified below¹²:

$$\Delta Y_t = \sum_{i=1}^{k-1} \Gamma \Delta Y_{t-i} + \Pi Y_{t-1} + \lambda D + \varepsilon_t \quad (19)$$

Where:

Y is vector of dependent variables (i.e. $\begin{pmatrix} i_w - i^* \\ i + i_w \end{pmatrix}$)

Γ , Π , and λ are matrix of parameters to be estimated. The long run relationship is determined by the rank of matrix Π , which is a product of α and β' (i.e. the adjustment and cointegrating vectors).

D represents the vector of independent variables which the dependent variable is regressed against. This includes a_0 (constant term), s_{it} (sensitivity of international investors), $stedres$ (i.e. short-term debt to reserves ratio), $stedares$ (i.e. short-term debt to residual reserves ratio), $fdgdp$ (i.e. fiscal deficit to gdp ratio) and any dummy variable. Δ is the change operator, while ε_t is the error term. The establishment of the existence of cointegrating vectors using the maximum eigen value and

¹² Upon establishment of the volatility of Indian foreign portfolio investment, Prabheesh (2013) estimated the discounted risk premium using the ARDL technique

trace test statistics would facilitate the test of the hypothesis of short run to long run adjustment and the cointegrating vectors.

Thus, the functional form of multivariate cointegration is as follows

$$\left(\frac{i-i^*}{1+i}\right) = f\left(\text{sii}, \frac{\text{sted}}{rr}, \frac{\text{fd}}{\text{gdp}}\right) \quad (20)$$

Transforming equation into logarithm equation 15 above becomes

$$\log\left(\frac{i-i^*}{1+i}\right) = a_0 + a_1 \log(\text{sii}) + a_2 \log\left(\frac{\text{sted}}{rr}\right) + a_3 \log\left(\frac{\text{fd}}{\text{gdp}}\right) + \varepsilon_t \quad (21)$$

$$\log\left(\frac{i-i^*}{1+i}\right) = a_0 + a_1 \log(\text{sii}) + a_2 \log\left(\frac{\text{sted}}{arr}\right) + a_3 \log\left(\frac{\text{fd}}{\text{gdp}}\right) + \varepsilon_t \quad (21.1)$$

The estimated reduced form discounted spread equations were:

$$\log \text{dspr} = a_0 + a_1 \log(\text{fpi}) + a_2 \log(\text{stedres}) + a_3 \log(\text{fdgdp}) + \varepsilon_t \quad (22)$$

$$\log \text{dspr} = a_0 + a_1 \log(\text{fpi}) + a_2 \log(\text{stedares}) + a_3 \log(\text{fdgdp}) + \varepsilon_t \quad (22.1)$$

Where:

- Dspr is the spread $\left(\frac{i-i^*}{1+i}\right)$
- Sii = fpi is the measure to capture sentiments of international investors (i.e. volatility of portfolio investment)
- sted/rr is the ratio is the ratio of short-term sovereign debt obligations to reserves
- sted/arr is the ratio is the ratio of short-term sovereign debt obligations to adjusted reserves, where adjusted reserves is actual reserves less FPI
- fd/gdp is fiscal deficit to GDP ratio and
- ε_t is the error term

The paper analyzed the stock of core reserves (i.e. actual reserves less adjusted quarterly portfolio investment) in addition to a separate analysis based on actual reserves using Nigerian data.

5.0 Empirical Results

This segment presents the descriptive statistic as well as the results of the empirical findings based on the estimation of the equations (22) and (22.1)

5.1 Descriptive Statistics and Unit Root

5.1.1 Descriptive Statistics

Summary statistics of the variables considered for risk premium equation using the actual reserves are shown in Table 2, while the summary statistics of variables for risk premium equation using the actual reserves are shown in Table 2a. The Jarque-Bera statistic indicates that the null hypothesis of normality is rejected for the logarithm values of discounted interest rate spread/premium (LDSPR) and fiscal deficit as a ratio of GDP (LFDGDP); while the null hypothesis of normality could not be rejected for foreign portfolio investment (LFPI) and ratio of short-term debt to external reserves as well as ratio of short-term debt to residual external reserves.

Table 2: Descriptive Statistics (Actual Reserves)

	LDSPR	LFDGDP	LFPI	LSTED_RE
Mean	-3.345900	-4.092614	5.787736	-3.984917
Median	-3.413131	-3.902545	5.680465	-3.998183
Maximum	-2.063523	-3.213540	8.819940	-1.826385
Minimum	-5.960179	-5.990789	2.833255	-7.834013
Std. Dev.	0.789206	0.750698	1.606783	1.723679
Skewness	-0.856780	-0.909260	0.108875	-0.459206
Kurtosis	4.812752	2.784798	2.095646	2.317417
Jarque-Bera	14.77809	7.964148	2.055021	3.109828
Probability	0.000618	0.018647	0.357897	0.211208
Sum	-190.7163	-233.2790	329.9009	-227.1402

Table 2a: Descriptive Statistics (Residual Reserves)

	LDSPR	LFDGDP	LFPI	LSTED_AR
Mean	-3.345900	-4.092614	5.787736	-3.891352
Median	-3.413131	-3.902545	5.680465	-3.984603
Maximum	-2.063523	-3.213540	8.819940	-1.783103
Minimum	-5.960179	-5.990789	2.833255	-7.461958
Std. Dev.	0.789206	0.750698	1.606783	1.621511
Skewness	-0.856780	-0.909260	0.108875	-0.321037
Kurtosis	4.812752	2.784798	2.095646	2.135916
Jarque-Bera	14.77809	7.964148	2.055021	2.752389
Probability	0.000618	0.018647	0.357897	0.252538
Sum	-190.7163	-233.2790	329.9009	-221.8071

5.1.2 Unit Root Test

As a starting point, the time series properties of the data were evaluated by adopting the Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) procedures for unit root test. The results as reported in Table 3 show that all the variables are stationary at first difference.

Table 3: Unit Root Test Results (Actual Reserves)

Variable	ADF (0)	ADF(1)	PP (0)	PP (1)
LDSPR	-1.292161	-8.959460*	-2.858988	-11.02280*
LFDGDP	-2.367539	-4.776235*	-1.787240	-4.349323*
LFPI	-1.172305	-6.334739*	-1.404593	-6.360871*
LSTED_RE	1.757566	-6.778355*	1.870506	-6.771807*
LSTED_AR	1.447163	-6.567720*	1.367062	-6.567720*

*Denotes rejection of H_0 at 1% significance level

Tables 4 and 4a indicate that the spread equation variables are weakly correlated with each other (except for foreign portfolio investment and ratio of short-term debt to reserves [-0.84] and ratio of short-term debt to residual reserves [-0.82]).

Table 4: Correlation Matrix (Actual Reserves)

	LDSPR	LFDGDP	LFPI	LSTED_RE
LDSPR	1.000000	0.340962	0.392743	-0.452459
LFDGDP	0.340962	1.000000	-0.077007	-0.043190
LFPI	0.392743	-0.077007	1.000000	-0.834449
LSTED_RE	-0.452459	-0.043190	-0.834449	1.000000

Table 4a: Correlation Matrix (Residual Reserves)

	LDSPR	LFDGDP	LFPI	LSTED_AR3
LDSPR	1.000000	0.340962	0.392743	-0.431671
LFDGDP	0.340962	1.000000	-0.077007	-0.031076
LFPI	0.392743	-0.077007	1.000000	-0.824234
LSTED_AR3	-0.431671	-0.031076	-0.824234	1.000000

5.3 Estimation of Cost of Default (C0)

Using the HP filter method, the study was able to generate the series on potential quarterly Nominal GDP and the output gap for the study period. Given that the Nigerian Banking crisis occurred immediately after the global economic crisis, the cumulative crisis period for the study was 2008 – 2010. Consequently, the study imposed the cumulative quarterly output loss between 2008Q1 and 2009Q4 (i.e. 52.80 per cent) and 2009Q1 – 2010Q4 (i.e. 32.22 per cent) as the maximum and minimum output losses for the entire study period. Table 5 illustrates the quarterly output losses due to the financial crisis between 2008Q1 and 2010Q4. The cumulative output loss between 2008Q1 and 2009Q4 (the height of the crisis) was 52.80 per cent will the cumulative output loss between 2009Q1 and 2010Q4 when the crisis was abating was 32.40 per cent. Therefore, the optimal reserve was calculated by considering the two ranges of output contraction i.e. 52.8 per cent and 32.40 per cent.

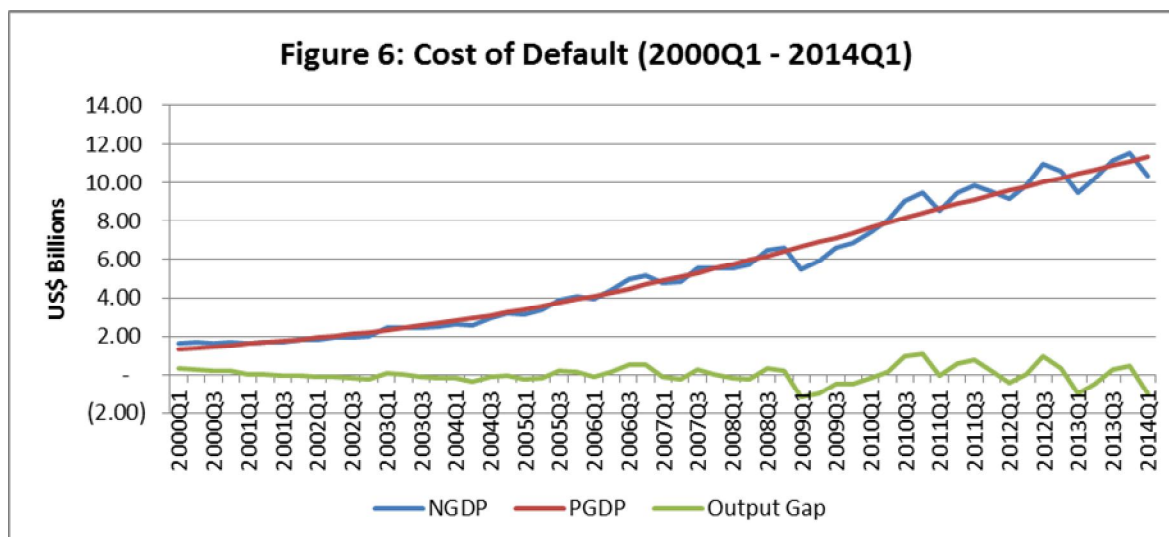


Table 5: Actual and Potential Output

Year/ Quarter	Actual Output	Potential Output	% Deviation
2008Q1	5,535,963.74	5,712,738.63	-3.19%
2008Q2	5,720,249.45	5,933,482.27	-3.73%
2008Q3	6,461,894.67	6,159,267.09	4.68%
2008Q4	6,578,221.42	6,390,089.96	2.86%
2009Q1	5,460,764.42	6,626,136.90	-21.34%
2009Q2	5,872,694.58	6,867,711.54	-16.94%
2009Q3	6,608,436.40	7,114,389.10	-7.66%
2009Q4	6,852,343.26	7,365,122.97	-7.48%
2010Q1	7,426,523.85	7,618,550.27	-2.59%
2010Q2	8,043,198.10	7,872,987.67	2.12%
2010Q3	9,055,632.86	8,126,631.79	10.26%
2010Q4	9,459,399.32	8,377,785.67	11.43%

5.4 Volatility of Foreign Portfolio Investment (FPI)

Unlike the approach of Prabheesh (2013), the paper estimated the volatility of foreign portfolio investment using weekly FPI (WFPI) data and a Generalised Autoregressive Conditional Heteroschedastic (GARCH) (1, 2) model. Figure 5, shows the weekly trend in FPI from January 21, 2013 to August 29, 2014. Table 5, which is the result of the GARCH (1, 2) model indicates that the conditional variance has a significant GARCH effect. The diagnostic tests on the model confirm the absence of serial correlation in the standardized squared residuals as well as the absence of GARCH effect on the residuals.

Figure 7: Trend in Foreign Portfolio Investment Returns

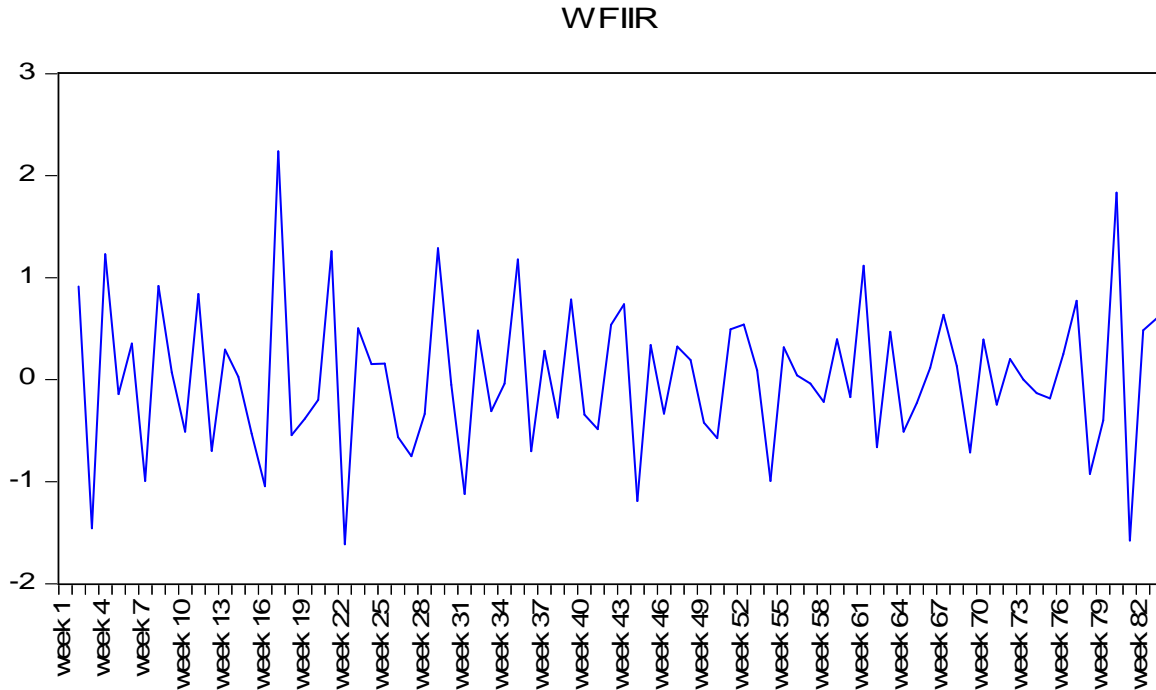


Table 6: GARCH (1, 2) results of Foreign Institutional Investment

$fii_t = 153.82 + 0.21fii_{t-1} + \varepsilon_t$ <p style="text-align: center;">(6.98)* (2.64)*</p>
$h_t = 10998.05 - 0.03\varepsilon_{t-1}^2 + 1.21h_{t-1} - 1.05h_{t-2}$ <p style="text-align: center;">(4.41)* (-2.80)* (28.26)* (-31.47)*</p>
Log likelihood = -203.15, LM_ARCH = 0.01 [0.96] $\chi^2 = 0.00[0.96]$

Note: * denotes significance at 1 per cent critical levels

Figures in parenthesis represent T-statistic, while those in square brackets represent level of significance

5.5 Johansen Cointegration

Given that all variables are integrated of order (1), the multi-variate cointegration technique of Johansen and Juselius (1992) was used to derive the equation for the risk premium that was used to measure the probability of default. The paper adopted an optimal lag length of 5, at which point the residuals of the VAR were found to be uncorrelated and homoscedastic (i.e. constant variance).

5.5.1 Analysis based on Actual Reserves

The trace and maximum eigen test statistics in Table 7 provides evidence that the null hypothesis of no cointegration can be rejected at the 5 per cent critical value for the model using actual

reserves, implying that there exists a set of cointegrating relationship among the four variables in the system.

Table 7: Johansen Cointegration Test based on Actual Reserves

Hypothesized No of CV(s)	Trace Statistic	P-value	Max-Eigen Statistic	P-value
None	71.26861	0.0007*	30.65420	0.0268*
At most 1	40.61441	0.0118*	22.82410	0.0422*
At most 2	17.79031	0.1057	13.40790	0.1182
At most 3	4.382411	0.3582	4.382411	0.3582

The normalized cointegrating coefficients with respect to LDSPR are given below in Table 7a.

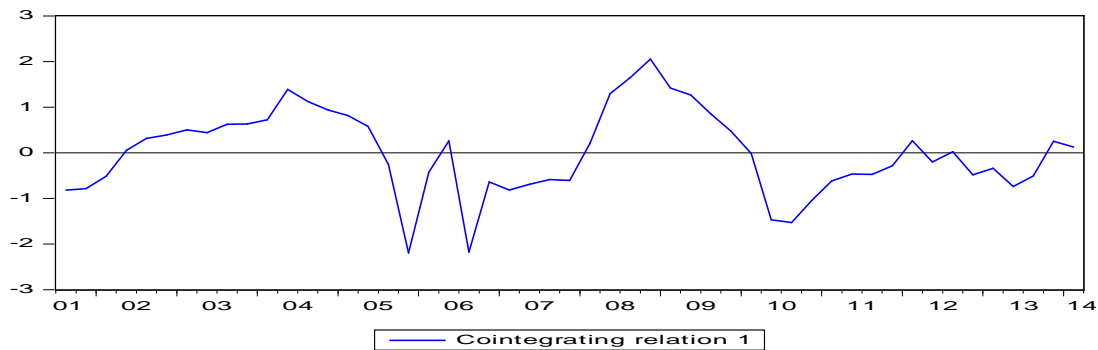
Table 7a: Long run coefficients of the cointegrating vectors – Actual Reserves

LDSPR	LFPI	LFDGDP	LSTED_RE	CONSTANT
	7.256688*	2.901351	3.560122	11.18279
	(2.25957)	(1.77248)	(2.21977)	(9.33945)

Note: Figures in parenthesis indicate standard error, *denotes 5 % significance level

The coefficients of the long-run cointegrating equation have the expected a priori signs but the result suggests that only one regressor (FPI) is statistically significant at 5 per cent. The relationship between volatility of Foreign Portfolio Investment (FPI) and spread is positive and highly significant. This tends to suggest that short-term capital flows reflects the risks inherent in the Nigerian economy. The cointegrating graph in figure 8 shows that the relationship among the variables is fairly stable across the sample range.

Figure 8: Cointegration Graph using actual reserves



From the estimated spread equation we derive the probability of default π by estimating $ldspr$, given as $\log(i - i^*/1 + i)$ which is then plugged in the equation below:

$$\pi = e^f / (1 + e^f)$$

$$f = \log(i - i^*/1 + i)$$

$$\ln\left(\frac{\pi}{1 - \pi}\right)_t = 11.18 + 7.26lfp_i + 2.90lfdgdp_i + 3.56lsted_re_i \quad (23)$$

$$\frac{\pi}{(1-\pi)} = \frac{i-i^*}{(1+i)}$$

The estimated average probability of default π is 5.63 per cent; with the maximum and minimum default probability values being 99.9 and 0.00 per cent, respectively. The time varying probability of default captures the sovereign risk of a country better than the traditional approach which assumes a default probability value of 50.00 per cent.

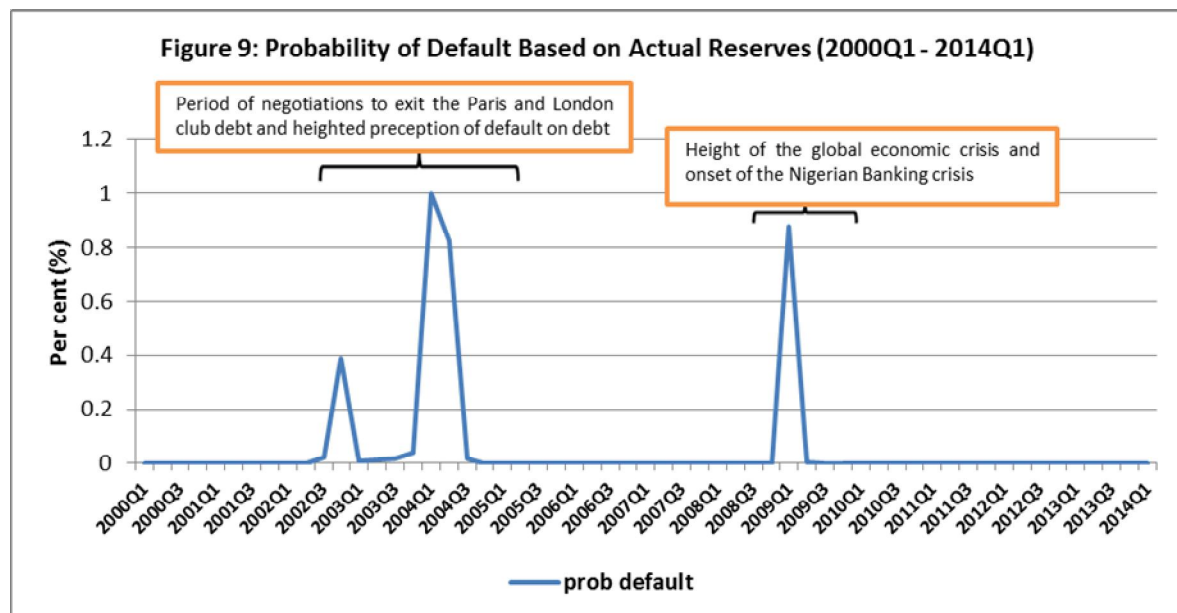
Considering the spread equation (16), π_R can be derived by differentiating it with respect to (res).

$$\frac{\partial^2 \pi}{\partial res} = \pi_R = -\pi(1-\pi) \frac{3.56}{res} < 0 \quad (24)$$

It shows that the change in probability of default due to a small accretion to external reserves is negative. In other words, the probability of default diminishes as a country accumulates more external reserves.

5.5.1.1 Probability of Default (π) – Actual Reserves

Figure 9, plots the estimated probability of default based on equation 17 (derived from actual reserves values) for the study period. The estimated average probability of default π is found to be 0.06 with maximum and minimum values of 99.9 and 0.00 per cent, respectively.



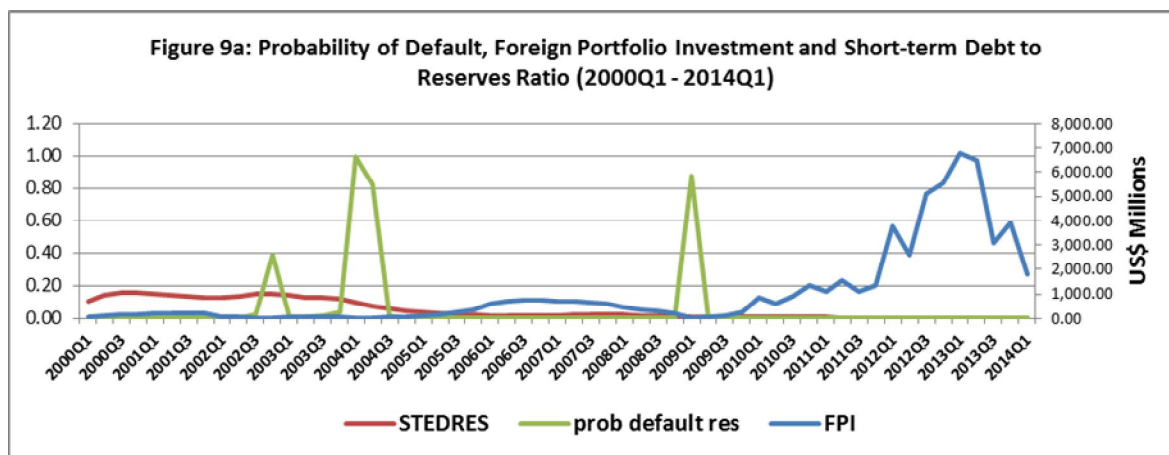


Figure 9a provides an explanation for the probability of default series observed in Figure 9. Between 2000q1 and 2002Q2, probability of default was close to zero. Within that period, FPI averaged US\$145.81 million per quarter.

Negotiations on the exit from the Paris and London club debt commenced in 2002 and lasted till 2004. This invariably increased concerns about possibility of default and loss of investments on the part of international investors, which invariably manifested in a decline in FPI and increase in the probability of default. Within the period 2002Q3 to 2004Q4, the average quarterly FPI dropped to US\$42.23 million, while the average probability of default increased to 23.25 per cent, reaching a peak of 99.9 per cent in 2004Q1.

Between 2005Q1 and 2008Q3, the average quarterly probability of default was close to zero on the back of sound economic fundamentals. This helped stimulate an increase in FPI within the period to a quarterly average of US\$497.13. The contagion effect of the global economic crisis which commenced in 2008Q3 hit the Nigerian economy between 2008Q4 and 2009Q1, which precipitated significant drop in the market capitalization of the Nigerian Stock Exchange (NSE) as well as the Nigerian banking sector crisis. Between 2008Q4 and 2009Q4, the joint crises prompted a decline in average quarterly FPI to US\$136.98 and an increase in average probability of default to 17.62 per cent. Within the period, quarterly FPI reached a minimum value of US\$23.64 million, while probability of default reached a maximum value of 87.9 per cent in 2009Q4.

Actions by the monetary and fiscal authorities in the face of the crises, coupled with good economic fundamentals, helped restore investor confidence in the economy. The devaluation of the Naira via the adjustment of the exchange rate midpoint helped moderate reserves depletion, while a wide interest rate spread helped attract FPI into the economy. As a result, between 2010Q1 and 2014Q1, the FPI and probability of default averaged US\$2.82 billion and 0.00 per cent, respectively.

5.5.2 Analysis of Residual/Adjusted Reserves

The maximum eigen value in Table 8, indicates no integration at the 5.0 per cent critical level. However, this is very marginal and could not be rejected at the 6.0 to 10.0 per cent critical levels. Furthermore, the trace statistic indicates cointegration of at least two equations. Consequently, it is safe to assume cointegration of at least 2 equations at the 6.0 per cent critical level, to facilitate the investigation of optimal reserves based on adjusted stock of foreign reserves.

Table 8: Johansen Cointegration Test based on Adjusted Reserves

Hypothesized No of CV(s)	Trace Statistic	P-value	Max-Eigen Statistic	P-value
None	68.97403*	0.0014	28.41955	0.0525
At most 1	40.55448*	0.0120	21.93722	0.0561
At most 2	18.61727	0.0828	14.15008	0.0920
At most 3	4.467188	0.3470	4.467188	0.3470

The normalized cointegrating coefficients with respect to LDSPR are given below in Table 8a.

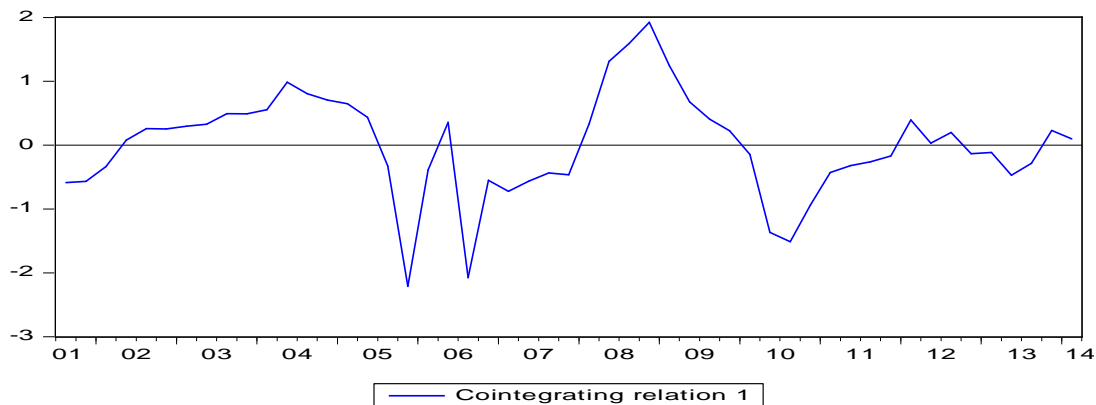
Table 8a: Long-run coefficient of the cointegrating vector – Adjusted Reserves

LDSPR	LFPI	LFDGDP	LSTED_AR	CONSTANT
	2.000296	1.007854	1.107000	4.773614
	(0.58048)*	(0.47750)*	(0.59659)**	(2.51673)**

Note: Figures in parenthesis indicate standard error, *denotes 5 per cent significance level and ** denotes 10 per cent significance level.

The coefficients of long run cointegrating equation have the expected a priori signs and the “t” statistic indicates two statistically significant regressors at the 5.00 per cent (i.e. fpi and fdgdp) and 10 per cent (i.e. sted_ar and constant) critical levels, respectively. In essence, we can conclude that the explanatory variables significantly explain the risk perception of foreign investors in the Nigerian economy. Again, the relationship between volatility of FPI and spread is positive and highly significant, which further confirms that short-term capital flows reflects the risks inherent in the economy. The cointegrating graph in figure 10 shows that the relationship among the variables is fairly stable across the sample range.

Figure 10: Cointegration Graph using adjusted reserves



From the estimated spread equation (17a), we derive the probability of default (π) by estimating $ldspr$, given as $\log(i - i^*/1 + i)$ which is then plugged in the equation below:

$$\pi = e^f / (1 + e^f)$$

$$f = \log(i - i^*/1 + i)$$

$$1\left(\frac{\pi}{1 - \pi}\right)_t = 4.77 + 2.00lfp_i + 1.01fdgdp_t + 1.11lstedres_t \quad (25)$$

$$\frac{\pi}{(1 - \pi)} = \frac{i - i^*}{(1 + i)}$$

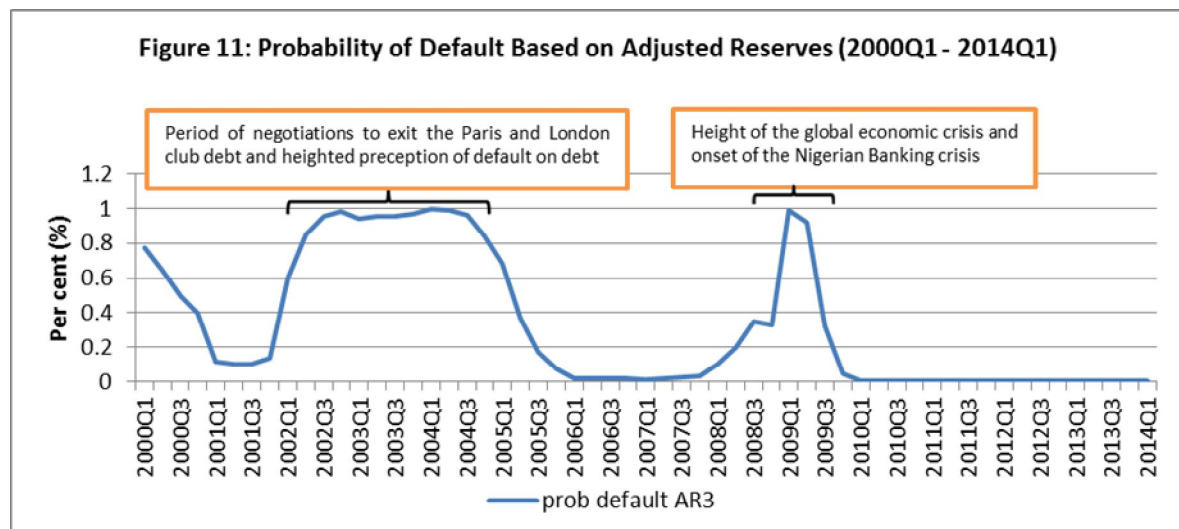
The estimated average probability of default π is 32.37 per cent; with the maximum and minimum default probability values being 99.81 and 0.00 per cent, respectively. In line with Prabheesh (2013), the time varying probability of default captures the sovereign risk of a country better than the traditional approach which assumes a default probability value of 50.00 per cent. Considering the spread equation (17a), π_R can be derived by differentiating it with respect to (res).

$$\frac{\partial^2 \pi}{\partial res} = \pi_R = -\pi(1 - \pi) \frac{1.11}{res} < 0 \quad (26)$$

It shows that the change in probability of default due to a small accretion to external reserves is negative. In other words, the probability of default diminishes as a country accumulates more external reserves.

5.5.2.1 Estimation of Probability of Default (π)

Figure 11, plots the estimated probability of default based on equation 17a (derived from adjusted reserves values) for the study period. As earlier discussed, this measure is germane for the study as it gives a fair perception of the ability of the economy to defend the international value of the Naira in the event of capital flight. The estimated average probability of default π is found to be 32.37 per cent with maximum and minimum values of 99.81 and 0.00 per cent, respectively.



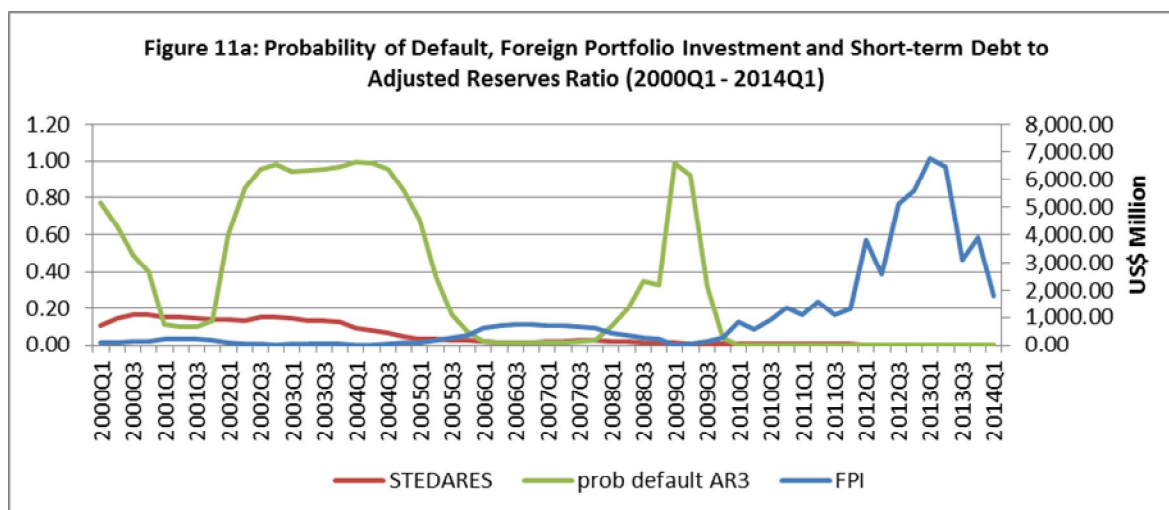


Figure 11a provides an explanation for the probability of default series observed in figure 11. A significant observation is that while the average FPI remained the same for the periods considered, the probability of default was significantly higher based on computations using adjusted /core reserves. The observed average probabilities were 42.01, 95.48, 13.88, 52.17¹³ and 0.06 per cent, for the periods 2000Q1 – 2002Q2, 2002Q3 – 2004Q4, 2005Q1 – 2008Q3, 2008Q4 – 2009Q4 and 2010Q1 – 2014Q1, respectively. The primary explanation for this is that computations based on adjusted reserves picked up on the impact of potential capital flight through the adjusted reserves and its interaction with short-term external debt (see Table 9).

Table 9: Periodic probability of default Actual & adjusted reserves

		Period				
		2000Q1 - 2002Q2	2003Q3 - 2004Q4	2005Q1 - 2008Q3	2008Q4 - 2009Q4	2010Q1 - 2014Q1
Probability of default	Actual Reserves	0.0%	23.25%	0.00%	17.62%	0.00%
	Adjusted Reserves	42.01%	95.48%	13.88%	52.17%	0.06%

5.6 Optimum Reserves

Following the estimation of π , π_R , C_0 and r we computed the optimum reserves for each period by substituting the relevant values into the optimal reserves equation (4).

$$[R^* = \frac{(1-\pi)}{\pi_R} + \frac{C_0}{r}].$$

The resulting period specific optimal reserves (based on actual and

adjusted reserves) are depicted in the charts below.

¹³ The maximum probability of default for the period 2008Q4 – 2009Q4, occurred at 2009Q4 reaching 99.18, while the maximum probability of default for the entire study period was attained in 2004Q1, reaching 99.81 per cent.

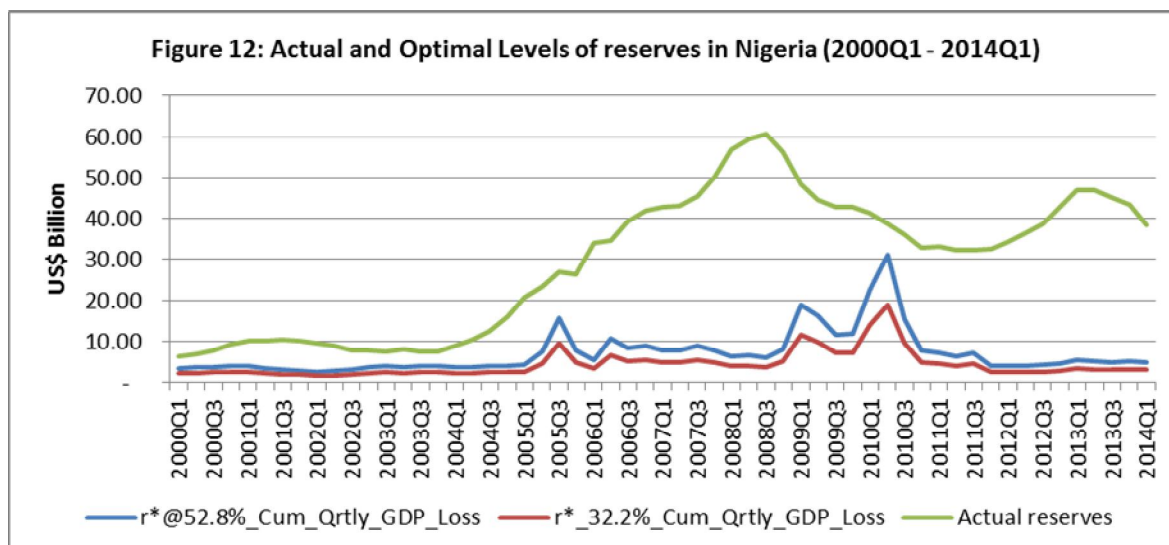


Figure 12 indicates that during the crisis period, the optimal core foreign exchange reserve for Nigeria was US\$31.22 billion. Intuitively, the Nigerian economy needed to have a minimum stock of US\$31.22 billion to be able absorb the potential output loss and cost of holding reserves associated with the severe impact of the global economic and Nigerian banking crisis from 2008Q1 – 2010Q4. A review of figure 12, suggests that the actual stock of Nigeria’s foreign reserves during and after the joint crises periods was adequate to absorb the impact of the crises. This however is not a reflection of the true health of reserves.

In order to obtain a more complete view of the health of the national reserves, the optimal reserves based on adjusted/residual reserves was used in the analysis (as earlier discussed). The result of the computation are depicted in figure 12a, which indicated that the optimal level of reserves required by the economy to adequately absorb the losses associated with the joint crises was US\$31.84 billion. Significantly, an examination of the adjusted reserves vis-à-vis the optimal level suggests that with the exception of 2010 Q1 & Q2, and 2013Q2, adjusted reserves has been below the optimal level required to absorb an impact of a severe crises like that witnessed between 2008Q1 and 2010Q4.

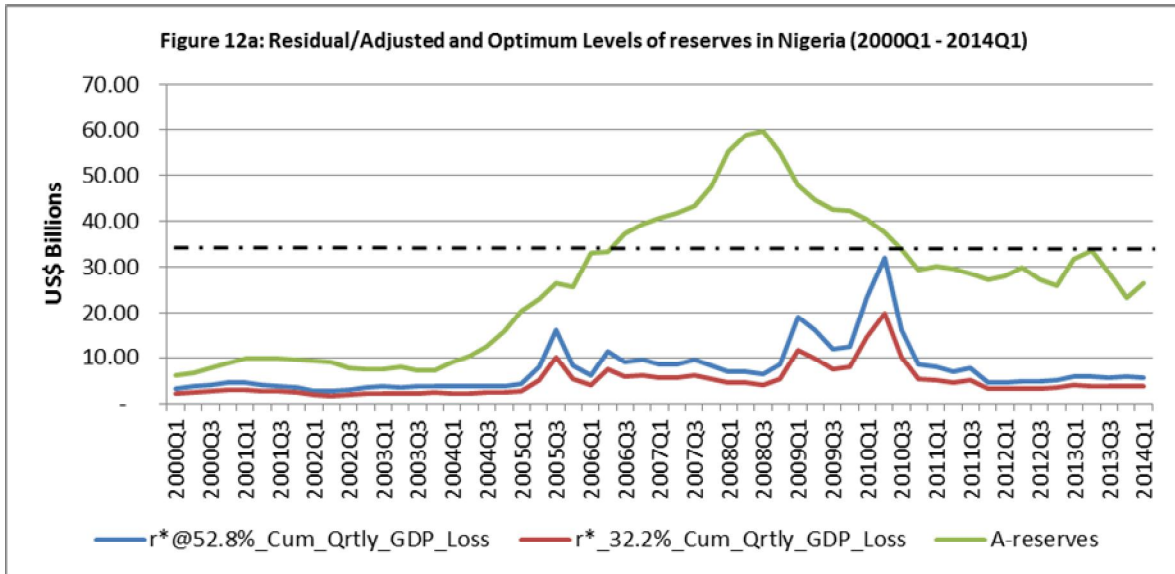
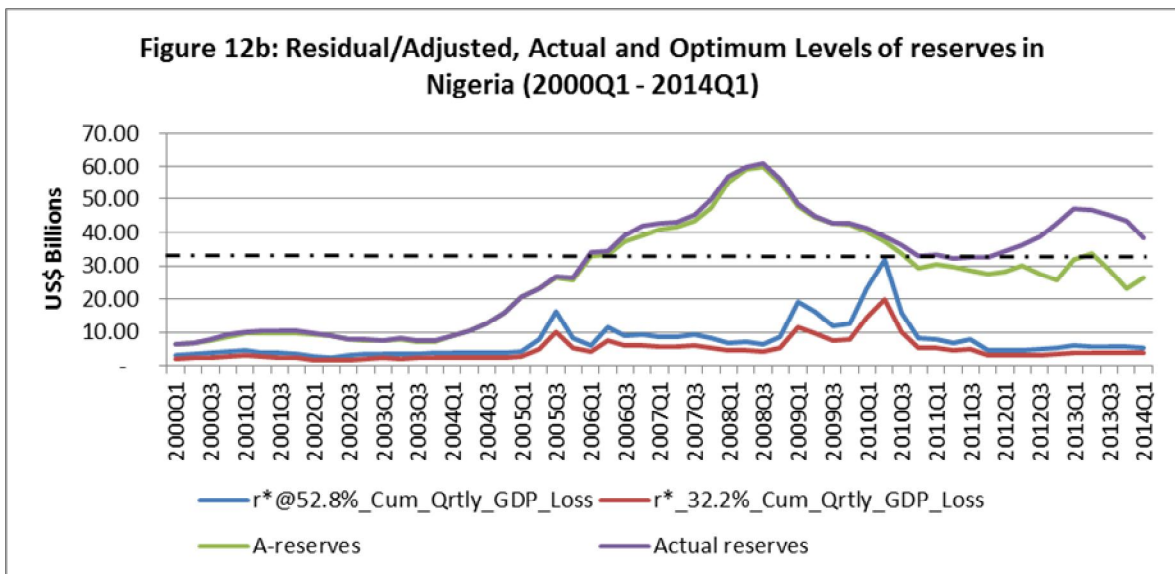


Figure 12b, further buttresses the point earlier made. It indicates that while actual reserves is above the optimal reserves level to absorb the impact of the joint crises between 2008Q1 and 2010Q4, the average core reserves available to the economy since the crises is however insufficient to absorb such a crisis if it occurs in the near future.



Consequently, based on the empirical findings of the paper, it is recommended that the Nigerian economy maintains a minimum core reserve of level of US\$32 billion. The reserves maintenance equation is thus represented as:

$$(FRML) = US\$32 \text{ billion} + \text{stock of FPI or HM into the economy} \quad (27)$$

Where:

FRML = Foreign reserves maintenance level,

FPI = Foreign portfolio investment into the Nigerian economy

HM = Hot Money invested in the Nigerian economy by foreign institutional investors.

6.0 Policy Recommendations and Conclusion

The primary finding of the paper suggests that given the severe impact of the joint crises (global economic crisis and Nigerian Banking Sector crisis) of between 2008Q1 and 2010Q4, Nigeria require a minimum core foreign reserves level of US\$32 billion to adequately absorb similar external shocks to the economy. Consequently, the foreign reserve maintenance (FRML) for the Nigerian economy is US\$32 billion (being the equivalent of 7.2 months of import cover¹⁴ at current price) plus the stock of foreign portfolio investments (FPI) currently within the economy. In view of this, the following policy recommendations are suggested.

1. Given that the current level of external reserves is about US\$39 billion; out of which US\$20 billion constitute foreign portfolio investments, it is imperative to build core reserves from the current level of US\$19 billion to the derived minimum level of US\$32 billion. This may be achieved by the following ways:
 - a. Block all foreign reserves leakages in the economy by ensuring that foreign exchange allocation and utilization are purely for genuine economic purposes. In doing this, the following areas are of great importance:
 - (i) Speculative demand should be curbed through effective monitoring of banks and building of a robust end-user intelligence management.
 - (ii) The Central Bank should initiate strategic engagement with the Federal Ministry of Finance (MoF) and the Nigeria Customs Service (NCS) to ensure that the seaports and borders are adequately secured against contraband and prohibited goods.
 - (iii) Implement an exchange rate policy that will reduce the current high exchange rate premium which provides arbitrage opportunities in the market.
 - (iv) Foreign payment through letters of credits is considered expensive. Management should consider replacing it with alternative payment modes like bills for collection, etc.
 - b. Curb excessive importation especially of goods and services that can be produced or rendered locally.
 - (i) In addition to the strict enforcement of the import prohibition list by relevant agencies of government, government should identify specific import goods which can be produced locally and provide incentives for small and medium scale enterprises to begin and/or increase the production of such goods.
 - (ii) The current policy on Rice importation and local production should be sustained and extended to other agricultural and agro-allied products.
 - c. Pursue fiscal consolidation at all levels of government.

¹⁴ This is based on average quarterly imports for the past four (4) years.

- d. Build fiscal buffers through the replenishment of the Excess Crude Account or the Sovereign Wealth Fund.
 - e. Revise the oil budget benchmark downward to allow for excess revenue that can be used to build fiscal buffers.
 - f. Measures should be taken to ensure that all exporters, especially in the oil and gas industry repatriate proceeds within stipulated period as provided in the guidelines.
2. Given that the interest rate spread is indicative of the sovereign risk of Nigeria, a sudden reduction in the domestic NTB rates will be adverse to FPI as the reduced spread will not generate sufficient compensatory premium for institutional investors to keep their investment in Nigeria's sovereign debt instruments. As such the paper recommends that reductions to NTB rates should be gradual and reflect changing fundamentals of the Nigerian economy.
3. The over dependence on the oil and gas sector in foreign revenue generation remains a point of concern, and the paper recommends the implementation of appropriate policies to facilitate the diversification of the Nigerian economy along areas of key comparative advantage (i.e. agriculture and small scale labour intensive manufacturing). This would help mitigate the impact of external shocks on the foreign exchange earnings base of the economy, and eventually help narrow the discounted risk premium as the economic fundamentals improve.
4. All the refineries should be fixed and be made to operate at their full capacity while other private sector interest are encouraged to venture into oil refining in Nigeria. The importation of refined oil should be discontinued.
5. The paper recommends the sustenance of supply side policies aimed at removing supply bottlenecks and improving the productive capacity of the Nigerian economy. This would further help improve the macroeconomic fundamentals of the economy and thus help facilitate a reduction in the compensatory premium required by foreign investors for absorbing risks associated with acquiring Nigeria's sovereign debt instruments.

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